

Motivations

A Simple Logic for Computation (Why Paradoxes are unavoidable and useful)

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- New computing paradigm
 - * Concurrent, distributed, live, interactive systems
- Reconfigurable “soft” hardware (FPGAs)
- Unify hardware and software
- Re-examine foundations of CS & CE
- Digital circuits
 - * Ideal, not physical
 - * Combinational, sequential, asynchronous
 - * Analysis & synthesis

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Logic(s) and Circuits

- 1-valued (Spencer-Brown, Kauffman, Bricken)
 - * Void/1st Distinction => (NoExist)/True
 - * Enough for most of classical logic
- 2-valued (Boole, Cantor, etc)
 - * Redundant, not minimal
 - * Express contradiction in arith, paradox in algebra
- 2-valued w imaginary (Varela, Kauffman)
 - * True, paradox values
- 3-valued w imaginary (Peirce, Lukasiewicz, etc)
 - * Hold simple paradox as a value
- 3x1-valued (James)
 - * Numbers up to exponentiation
- 4-valued (Kauffman, Hellerstein, Shoup)
 - * Symmetrical, from 2-valued
 - * Hold full contradiction, paradox as values

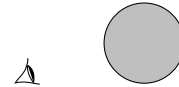
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First Distinction

- First Thing -- Simplicity Itself
- Pure difference, discrimination
- Pure form, no interpretation yet
- Smallest possible, unavoidable step from the Void
- Both object (True) and action (Invert)



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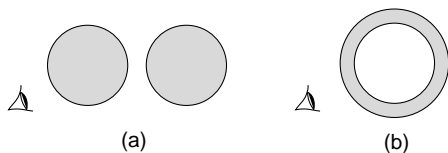
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Second Distinction

- Two possibilities
 - a) outside - symmetrical, cardinality
 - b) inside - asymmetrical, ordinality
- Allows first contradiction, paradox

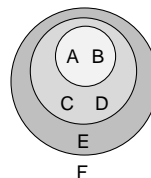
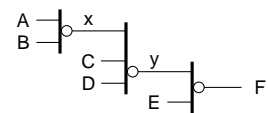


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Combinational Logic is Timeless



$$F = (((A \vee B) \vee C \vee D) \vee E)$$

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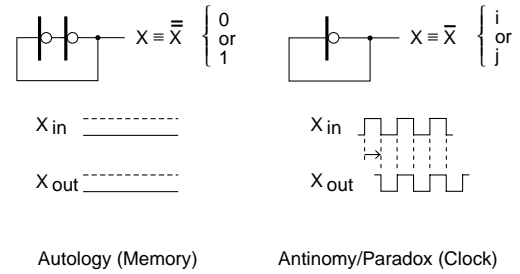
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Classic Paradoxes

- Epimenides (logic)
 - * "I am lying." ("This statement is False.")
- Russell (set theory)
 - * The set of all sets not containing themselves.
- the Barber (set theory or logic)
 - * Barber shaves all men who don't shave themselves.
- Berry (number theory)
 - * The smallest number not specifiable in less than N words.
- Gödel (logic/number/proof theory)
 - * "This statement is unprovable."
- Turing (computability)
 - * "This machine doesn't halt."

Self-Reference - the Origin of Time



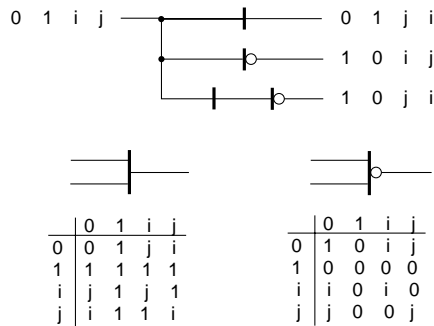
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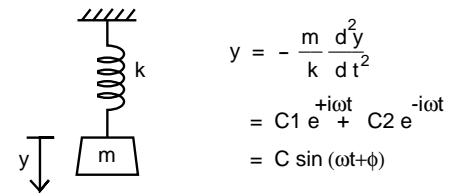
4-Valued Logic Holds Paradox



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Classical Self-Reference



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i : Logic $:: \sqrt{-1}$: Reals

Logic

$$x = \sim x$$

$$x = i \text{ or } j$$

$$0, 1, i, j \oplus$$

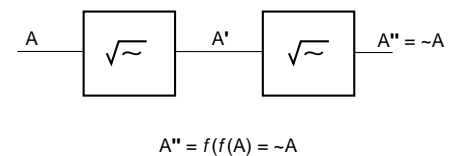
Numbers

$$x = -1/x \quad (x^2 = -1)$$

$$x = \sqrt{-1} = i$$

$$1, -1, i, -i \cdot$$

Square Root of NOT



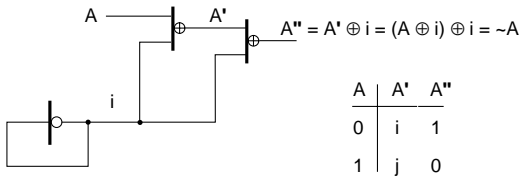
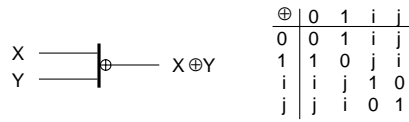
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Square Root of NOT Circuit

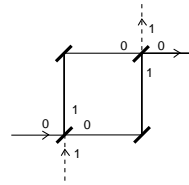


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Linking Two Sqrt(NOT)s - interference



$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

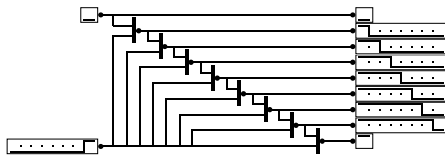
$$\begin{array}{c|c|c} a & b & n \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{array} \oplus \begin{array}{c|c|c} a' & b' & n' \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{array} \Rightarrow \begin{array}{c|c|c} a & ba' & b' & n' \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{array} \Rightarrow \begin{array}{c|c|c} a & b' & n' \\ \hline 0 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & -2 \\ 1 & 1 & 0 \end{array}$$

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Eighth Root of NOT

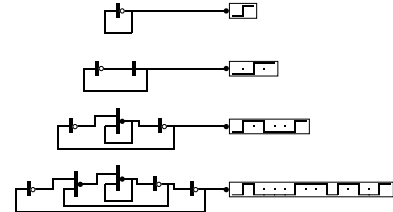


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Higher-Order Logic Values

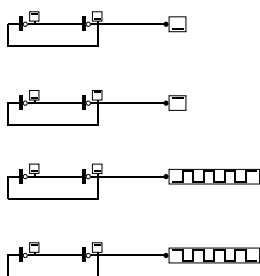


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Memory & Superposition

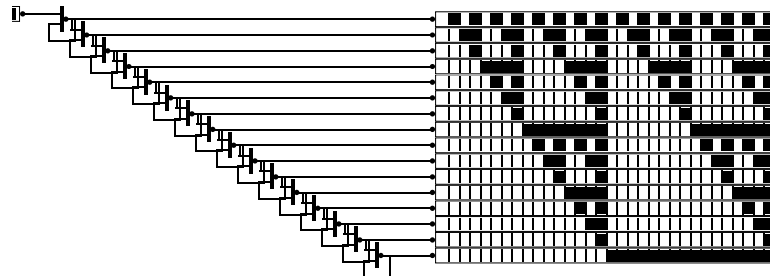


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Summary

- Self-reference generates Paradox
- 4-valued logic holds simple Paradox (0, 1, i, j)
 - * 4 values timeless \Leftrightarrow 2 values timely
 - * Higher-order paradoxes $\Rightarrow 2^{2^n}$ values
- Time not required/generated until Self-Reference
- Paradox is unavoidable, necessary, useful
 - * Recursion: bridge from finite to infinite forms
 - * Looping, asynchronous circuits
 - * Quantum mechanics, quantum computing

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About Logic 'Values'

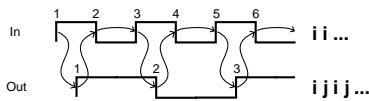
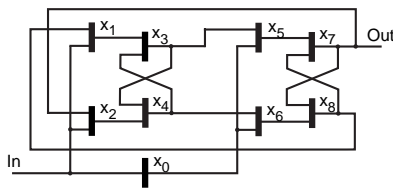
- Each circuit (expression) draws a distinction in the set of logic values of its constituents. If that subset is *empty*, then the circuit represents a *new logic 'value'*.
- A new (higher-order) logic 'value' is *not* equiv to simultaneous prior values (e.g. "both F and T"), but is *something entirely new*.
- Paradoxical self-reference can be created at any level, and thus *the hierarchy of logic 'values' is unbounded*.

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Binary Counter



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